

## General history of Logs and their function

A log is essentially the inverse of a power function, and can be applied to any number base for example:

$$2^3 = 8 \text{ so } \log_2 8 = 3$$

In other words the log function tells you what power you need to raise 2 to in order to get 8. The subscript number after log tells you the number base you are working to. Here are some other examples:

$$5^5 = 3125 \text{ so } \log_5 3125 = 5$$

$$10^3 = 1000 \text{ so } \log_{10} 1000 = 3$$

Outside of pure mathematics, only two log bases are in common use:

- Log to the base of 10,  $\log_{10}$  or ,more frequently simply log with the base omitted.
- Log the base e, where e is a non-recurring decimal (like pi) taking an approximate value of 2.7182818285 this is usually called the natural log base and is written ln.

When working with decibels, we always work to log base 10, commonly written as simply log.

Originally logs were useful in multiplication of large numbers since the sum (adding together) of the logs of two numbers is equal to the log of the product (multiplication) of those two numbers:

$$\log(a \times b) = \log a + \log b \text{ so } a \times b = 10^{\log a + \log b}$$

This may seem like a very complex way of doing a simple calculation, but in the days before calculators, logs were published in books called log tables and this process made complex calculations much faster for someone used to using the tables.

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## basic logarithm algebra

$$a = \log b \text{ so } 10^a = b$$

this is the basic definition of a log function.

$$\log ab = \log a + \log b$$

$\log ab$  is shorthand for  $\log(a \times b)$

$$\log a/b = \log a - \log b$$

$\log a/b$  is shorthand for  $\log(a \div b)$

$$\log b^c = c \log b$$

remember that  $\sqrt{a}$  is equivalent to  $a^{1/2}$  so  $\log \sqrt{b}$  is  $1/2 \log b$

This simple algebra should be enough for dealing with decibel logs.